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Tittle : **Free and Forced Convective three Dimensional Flow with Heat and Mass Transfer**

By

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DECLARATION

I Pubali Saikia bearing the Roll no-MAT-03/23, hereby declare that this dissertation entitled, " FREE AND FORCED CONVECTIVE WITH HEAT AND MASS TRANSFR" was carried out by me under the supervision of my guide Dr. Jugal Khargharia professor of the department of mathematics ,Mahapurusha Srimanta Sankaradeva viswavidyalaya , Nagaon. .The study and recommendations drawn are original. The data and facts stated in this survey are correct to the best of my knowledge.

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FREE AND FORCED CONVECTIVE WITH HEAT AND MASS TRANSFER

INTRODUCTION

The problem of three dimensional heat and mass transfer flow has been the object of extensive research due to its possible applications in many branches of science and technology . such problems are observed in buoyancy –induced motions in the atmosphere ,in bodies of water , quasi –solid bodies such as earth and soon . in nature and industrial applications , many transport processes exist where the transfer of heat and mass takes place simultaneously as a result of combined buoyancy effects of thermal diffusion and diffusion of chemical species industries . in addition the phenomenon of heat and mass transfer is also encountered in chemical species industries . in the light of this fact , a series of investigations have been made by (Raptis et al. 1981), (raptis and kafousias 1982).(Acharya et al. 2000) have analyzed the effect of free convection and mss transfer in steady flow through porous medium with constant suction in the presence of magnetic field .Moreover, (Ahmed and Ahmed 2004) have extend the problem studied by (Achary et al. 2000) to the unsteady case by considering a uniform motion of the plate .Very Recently ,(Ahmed and ahmed 2008) investigated the effect of periodic suction velocity and permeability on the heat transfer flow of an incompressible viscous dissipative fluid through a porous medium .

Many research workers are doing investigation of the problem of laminar flow control due to its importance in the field of aeronautical engineering ,in view of its application to reduce drag and hence the vehicle power requirement by a substantial amount. Initially this subject has been developed by (Lachmann 1961). Theoretical and experimental investigations have shown that the transition from laminar to the turbulent flow , which causes the drag coefficient to increase , may be prevented by suction of the fluid and heat and mass transfer from boundary to the wall .

To obtain any desired reduction by increasing suction alone is uneconomical as the energy consumption of the suction pump will be more. Therefore ,the method of " cooling of the wall " in controlling the laminar flow together with application of suction has become more useful and hence received the attention of many researchers .

(Singh et al 1978) investigated the effect on wall shear stress and heat transfer of the flow caused by the periodic suction velocity perpendicular to the flow caused by the periodic suction velocity perpendicular to the flow direction when the difference between the wall temperature and the free stream temperature gives rise to buoyancy force in direction of the free stream . The effect of the porous medium on the three –dimensional coquette flow with transpiration cooling . Futher (Singh et el 2007) have analysed the effects of periodic sunction velocity on three –dimensional viscous fluid with heat and mass transfer .

1.2 OBJECTIVE .

The purpose of the present paper is to study the effects of heat and mass transfer on the steady three dimensional flow of a viscous incompressible fluid along a steadily moving porous vertical plate subjected to a transverse sinusoidal suction velocity .

1.3 BASIC EQUATIONS :

Let us introduce a coordinates system with the plate lying vertically on XZ plane , such that the X-axis is oriented in the direction of buoyancy force and the Y-axis is perpendicular to the plane of the plate and directed into the fluid which is flowing laminarly with the free stream velocity U .The transverse sinusoidal suction velocity distribution is assumed to be of the form :

$$\bar{v}_w(\bar{z}) = -v_0(1 + \varepsilon cos\frac{\pi\bar{z}}{L}) \qquad \rightarrow 1.1$$

Where <<1 and L is the wave length of the periodic suction . All physical quantities are independent of $\bar{x}$ for this problem of fully developed laminar flow .

The governing equation of heat and mass transfer flow are given by continuity Equation :

$$\frac{\partial\bar{v}}{\partial\bar{y}} + \frac{\partial\bar{w}}{\partial\bar{z}} = 0 \qquad \rightarrow 1.2$$

Momentum Equation :

$$\bar{v}\frac{\partial\bar{u}}{\partial\bar{y}} + \bar{w}\frac{\partial\bar{u}}{\partial\bar{z}} = g\beta(T - T_\alpha) + g\beta(C - C_\alpha) + v(\frac{\partial^2\bar{u}}{\partial\bar{y}^2} + \frac{\partial^2\bar{u}}{\partial\bar{z}^2}) \qquad \rightarrow 1.3$$

$$\bar{v}\frac{\partial\bar{u}}{\partial\bar{y}} + \bar{w}\frac{\partial\bar{u}}{\partial\bar{z}} = -\frac{1}{\rho}\frac{\partial\bar{p}}{\partial\bar{y}} + v(\frac{\partial^2\bar{v}}{\partial\bar{y}^2} + \frac{\partial^2\bar{v}}{\partial\bar{z}^2}) \qquad \rightarrow 1.4$$

$$\bar{v}\frac{\partial\bar{w}}{\partial\bar{y}} + \bar{w}\frac{\partial\bar{w}}{\partial\bar{z}} = -\frac{1}{\rho}\frac{\partial\bar{p}}{\partial\bar{y}} + v(\frac{\partial^2\bar{w}}{\partial\bar{y}^2} + \frac{\partial^2\bar{w}}{\partial\bar{z}^2}) \qquad \rightarrow 1.5$$

Energy Equation :

$$\bar{v}\frac{\partial \bar{T}}{\partial y}+\bar{w}\frac{\partial \bar{T}}{\partial \bar{z}}=\alpha(\frac{\partial^2 \bar{T}}{\partial \bar{y}^2}+\frac{\partial^2 \bar{T}}{\partial \bar{z}^2}) \qquad \rightarrow 1.6$$

Species Concentration Equation :

$$\bar{v}\frac{\partial \bar{C}}{\partial \bar{y}}+\bar{w}\frac{\partial \bar{C}}{\partial \bar{z}}=D\left(\frac{\partial^2 \bar{C}}{\partial \bar{y}^2}+\frac{\partial^2 \bar{C}}{\partial \bar{z}^2}\right) \qquad \rightarrow 1.7$$

The boundary conditions of the problem are :

$$\bar{y}=0: \bar{u}=\bar{V}, \bar{v}=\bar{v}_w, \bar{w}=0, \bar{T}=T_{W\,,}\ \bar{C}=C_w$$

$$\bar{y}\rightarrow\propto: \bar{u}=\bar{U}, \bar{v}=-v_{0,}\ \bar{w}=0, \bar{T}=T_{\infty\ ,\bar{P}=P_{\infty},\ \ \bar{C}=C_\infty}$$

Let us introduce the following non –dimensional variables :

$$y=\frac{\bar{y}}{L}, z=\frac{\bar{z}}{L}, u=\frac{\bar{u}}{v_0}, v=\frac{\bar{v}}{v_0}, w=\frac{\bar{w}}{w_0}, p=\frac{\bar{p}}{\rho V^2}, U=\frac{\bar{U}}{v_{0,}}$$

$$\theta=\frac{\bar{T}-\bar{T}_m}{\bar{T}_{w-}\bar{T}_\alpha}\ , C=\frac{\bar{C}-\bar{C}_m}{\bar{C}_w-\bar{C}_\alpha}, V=\frac{\bar{V}}{v_0}, p=\frac{\bar{p}}{\rho(v/L)^2}, p_\alpha=\frac{\bar{p}_\alpha}{\rho(v/L)^2}\ ,$$

Where subscripts w =conditions on the wall and α =free stream conditions ,

Grashof number for heat transfer (Gr):

This number usually occurring in free convection problem .This gives the relative importance of buoyancy forces to the viscous forces in the velocity boundary layer and it can be defined as

$$Gr=\frac{Lg\beta(\bar{T}_w-\bar{T}_\alpha)}{{v_0}^2}$$

Grashof number for Mass Transfer (Gm):

This number usually occurring in free convestion problem .when the effect of mass transfer is also considered . this number in out can be defined as

$$Gr=\frac{Lg\bar{\beta}(\bar{C}_w-\bar{C}_\alpha)}{v_0^{2}}$$

Prandit number (Pr):

The physical interpretation of this number provides a measure of the relative effectiveness of momentum and energy transport by difference in the velocity and thermal boundary layers , respectively usually the relative growth of the velocity and thermal boundary $layers$ it is defined as

Pr $=$ / where / Cp

Reynolds number (Re):

The physical interpretation of Reynolds number is the ratio of the inertia forces to the viscous forces in the velocity boundary layer. It is defined as

Re$=v_0 L/$

Schmidt number (S):

The physical interpretation of the Schmidt number provides a measure of the relative effectiveness of momentum and mass transport by diffusion in the velocity and concentration boundary layers , respectively. It is defined as

S= $/_D$

In view of the above non-dimensional quantities , equations become:

$$\frac{v_0}{L}\frac{\partial v}{\partial y}+\frac{v_0}{L}\frac{\partial w}{\partial z}=0$$

$$\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \qquad \rightarrow 1.7$$

Again

$$\frac{v_0^2}{L} v \frac{\partial u}{\partial y} + \frac{v_0^2}{L} w \frac{\partial u}{\partial z} = g\beta(\bar{T}_w - T_\infty)\theta + g\bar{\beta}(\bar{C}_W - \bar{C}_\alpha)C + \frac{\vartheta V_0}{L}\left(\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right),$$

$$\Rightarrow v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = \frac{Lg\beta(\bar{T}_w - \bar{T}_\alpha)}{v_0^2}\theta + \frac{Lg\bar{\beta}(\bar{C}_W - \bar{C}_\alpha)}{v_0^2}C + \frac{\vartheta}{v_0 L}\left(\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right)$$

$$\Rightarrow v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = Gr\theta + GmC + \frac{1}{Re}\left(\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) \rightarrow 1.7$$

Again $\frac{v_0^2}{L} v \frac{\partial w}{\partial y} + \frac{v_0^2}{L} w \frac{\partial w}{\partial z} = -\frac{1}{p}\rho\left(\frac{v}{L}\right)^2 \frac{\partial P}{\partial Z}\frac{1}{L} + \frac{\vartheta v_0}{L^2}\left(\frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right)$

$$\Rightarrow v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} = -\left(\frac{v}{v_0 L}\right)^2 \frac{\partial \mathrm{P}}{\partial \mathrm{Z}} + \frac{\mathrm{v}}{\mathrm{v_0 L}}\left(\frac{\partial^2 \mathrm{w}}{\partial \mathrm{Y}^2} + \frac{\partial^2 \mathrm{w}}{\partial \mathrm{z}^2}\right)$$

$$\Rightarrow v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} = -\frac{1}{Re^2}\frac{\partial p}{\partial y} + \frac{1}{Re}\left(\frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right),$$

Again

$$\frac{v_0^2}{L} v \frac{\partial w}{\partial y} + \frac{v_0^2}{L} w \frac{\partial w}{\partial z} = -\frac{1}{\rho}\rho\left(\frac{v}{L}\right)^2 \frac{\partial p}{\partial z}\frac{1}{L} + \frac{\mu v_0}{L^2}\left(\frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right),$$

$$\Rightarrow v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} = -\left(\frac{v}{v_0 L}\right)^2 \frac{\partial p}{\partial z} + \frac{v}{v_0 L}\left(\frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right),$$

$$\Rightarrow v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} = -\frac{1}{Re^2}\frac{\partial p}{\partial z} + \frac{1}{Re}\left(\frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right),$$

$$\Rightarrow v_0 v\frac{\partial}{\partial y}[(\bar{T}_w - \bar{T}_\infty)\theta + \bar{T}_\infty]\frac{1}{L} + v_0 w\frac{\partial}{\partial y}[(T_w - T_\infty)\theta + T_\infty]\frac{1}{L}$$

$$= \frac{\alpha}{\mathrm{L}^2}\left(\frac{\partial^2}{\partial \mathrm{Y}^2} + \frac{\partial^2}{\partial \mathrm{Z}^2}\right)[(\bar{\mathrm{T}}_\mathrm{w} - \bar{\mathrm{T}}_\infty)\theta + \bar{\mathrm{T}}_\infty],$$

$$\Rightarrow \frac{v_0(T_w - T_\infty)}{L} v\frac{\partial \theta}{\partial Y} + \frac{v_0(T_w - T_\infty)}{L} w\frac{\partial \theta}{\partial Y} = \frac{\propto(\bar{\mathrm{T}}_\mathrm{w} - \bar{\mathrm{T}}_\infty)}{\mathrm{L}^2} + \left[\frac{\partial^2 \theta}{\partial \mathrm{Y}} + \frac{\partial^2 \theta}{\partial \mathrm{Z}}\right],$$

$$v\frac{\partial \theta}{\partial y} + w\frac{\partial \theta}{\partial z} = \frac{1}{Re\ Pr}\left(\frac{\partial^2 \theta}{\partial y^2} + \frac{\partial^2 \theta}{\partial z^2}\right),$$

$$v\frac{\partial C}{\partial y}+w\frac{\partial C}{\partial z}=\frac{1}{Re\,S}\left(\frac{\partial^2 C}{\partial Y^2}+\frac{\partial^2 C}{\partial Z^2}\right),$$

The corresponding boundary conditions to solve these equations are :

$$y=0: u=V, v(z)=-(1+\varepsilon cos\pi z), w=0, \theta=1, c=1$$

$$y\to\infty: u\to U, v\to -1, w\to 0, \theta\to 0, C\to 0, p\to p$$

1.4 METHOD OF SOLUTION:

When the amplitude ,$\varepsilon(<<1)$ of the suction velocity is small ,we assume the solutions the non –linear differential equations of the form:

$$F(v,z)=F_0(v)+\quad F_1(y,z)+o(\quad ^2)$$

Where F stands for u ,v ,w ,p, or $\dot{C}$

With $p_{0=p_\infty}$,$w_{0=0}$.

Substituting the equations into the equations to we get

$$\frac{\partial}{\partial y}(v_0+\varepsilon v_1)+\frac{\partial}{\partial z}(w_0+\varepsilon w_1)=0$$

$(v_0+\varepsilon v_1)\frac{\partial}{\partial y}(u_0+\varepsilon u_1)+(w_0+\varepsilon w_1)\frac{\partial}{\partial z}(u_0+\varepsilon u_1)=Gr(\theta_0+\varepsilon\theta_1)+$
$Gm(C_0+\varepsilon C_1)+\frac{1}{Re}\left[\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right](u_0+\varepsilon u_1)$

$(v_0+\varepsilon v_1)\frac{\partial}{\partial y}(v_0+\varepsilon v_1)+(w_0+\varepsilon w_1)\frac{\partial}{\partial z}(v_0+\varepsilon v_1)=-\frac{1}{Re^2}\frac{\partial}{\partial y}(p_0+\varepsilon p_1)$
$+\frac{1}{Re}\left[\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial y^2}\right](v_0+\varepsilon v_1)$

$(v_0+\varepsilon v_1)\frac{\partial}{\partial y}(w_0+\varepsilon w_1)+(w_0+\varepsilon w_1)\frac{\partial}{\partial z}(w_0+\varepsilon w_1)=-\frac{1}{Re^2}\frac{\partial}{\partial y}(p_0+\varepsilon p_1)$
$+\frac{1}{Re}\left[\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial y^2}\right](w_0+\varepsilon w_1)$

$(v_0+\varepsilon v_1)\frac{\partial}{\partial y}(\theta_0+\varepsilon\theta_1)+(w_0+\varepsilon w_1)\frac{\partial}{\partial z}(\theta_0+\varepsilon\theta_1)=-\frac{1}{RePr}\left[\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right](\theta_0+$
$\varepsilon\theta_1)$

$(v_0 + \varepsilon v_1)\frac{\partial}{\partial y}(C_0 + \varepsilon C_1)+(w_0 + \varepsilon w_1)\frac{\partial}{\partial z}(C_0 + \varepsilon C_1) = -\frac{1}{ReS}\left[\frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right](c_0 + \varepsilon c_1)$

When $\varepsilon = 0$ the problem reduces to the two –dimensional case and hence the equation reduce to the following ordinary differential equations :

$$\dot{v}_0 = 0$$

$$v_0\dot{u}_0 = Gr\vartheta_0 + GmC_0 + \frac{1}{Re}\ddot{u}_0$$

$$v_0\dot{\theta}_0 = \frac{1}{RePr}\ddot{\theta}_0$$

$$v_0\dot{C}_0 = \frac{1}{ReS}C_0$$

With corresponding boundary conditions :

$$y = 0: u_0 = V, v_0 = -1, \theta_0 = 1, C_0 = 1$$

$$y \to \infty: u_0 \to U, v_0 \to -1, \theta_0 \to 0, C_0 \to 0$$

Now, we have to solve the equations under the boundary conditions as follows:

$$v_0 = -1$$

$$\Rightarrow (D^2 - v_0RePrD)\theta_0 = 0, D = \frac{d}{dy}$$

Let us take the solution of y is

$\theta_0(y) = C_1e^{-RePry} + C_2$,where C_1 and C_2 are the constants.

At $y \to \infty, \theta_0 = 0 \Rightarrow C_2 = 0$

And $y = 0, \theta_0 = 1 \Rightarrow C_1 = 1$

$$\therefore \theta_0(y) = e^{-RePry}$$

$\Rightarrow (D^2 - v_0\text{Re SD})C_0 = 0, D \equiv \frac{d}{dy}$

Let us take the solution of $C_0(y)$ is

$C_0(y) = C_3 e^{-Re\,Sy} + C_4$, where C_3 and C_4 are the constants .

At $y \rightarrow 0, C_0(y) = 0 \Rightarrow C_4 = 0$

And $y = 0, C_0(y) = 1 \Rightarrow C_3 = 1$

$$\therefore C_0(y) = e^{-Re\,Sy}$$

$\Rightarrow (D^2\text{-}v_0\mathrm{Re\ D})\mathrm{u}_0 = -\mathrm{Re}(\mathrm{Gr}\theta_0 + \mathrm{GmC}_0), \mathrm{D} \equiv \frac{\mathrm{d}}{\mathrm{dy}} = -(\mathrm{Gre}^{-\mathrm{Re\ Pry}} + \mathrm{Gme}^{-\mathrm{ReS\ y}})$

Let us take the solution of $u_0(\mathrm{y})$ is

$u_0(\mathrm{y}) = \mathrm{C}_5\mathrm{e}^{-\mathrm{Re\ y}} + \mathrm{C}_6 + \mathrm{PI}$, where C_5 and C_6 are the constants .

The Particular integral (PI) for different values of Pr and S is as follows :

$$\mathrm{PI}=\frac{-Re}{D^2+ReD}[\mathrm{Gre}^{-\mathrm{Re\ Pr\ y}} + \mathrm{Gme}^{-\mathrm{Re\ S\ y}}]$$

$$=-\frac{ReGre^{-Re\,\mathrm{Pr}\,y}}{Re^2Pr^2-Re^2Pr} - \frac{\mathrm{ReGme}^{-\mathrm{Re\ Sy}}}{\mathrm{Re}^2\ \mathrm{S}^2-\mathrm{Re}^2\mathrm{S}}$$

$$=-\frac{Gre^{-Re\,\mathrm{Pr}\,y}}{Re(Pr^2-Pr)} - \frac{\mathrm{Gme}^{-\mathrm{Re\ S\ y}}}{\mathrm{Re}(\mathrm{S}^2-\mathrm{S})} \quad \text{for S}\neq 1,\text{Pr}\neq 1$$

When S$\neq$ 1 ,Pr =1 , then we have

$$\mathrm{PI} =-\frac{yReGre^{-Re\,y}}{2D+Re} - \frac{Gme^{-Re\,S\,y}}{Re(S^2-S)} = -\frac{yReGre^{-Re\,S\,y}}{(-2Re+Re)} - \frac{Gme^{-Re\,S\,y}}{Re(S^2-S)}$$

$$=\mathrm{y}Gre^{-Re\,y} - \frac{Gme^{-Re\,S\,y}}{Re(S^2-S)}$$

When S =1 ,Pr$\neq$1 , then we have

$$\mathrm{PI} = -\frac{yReGme^{-Rey}}{2D+Re} - \frac{Gme^{-ReSy}}{Re(S^2-S)} = -\frac{yReGre^{-Rey}}{(-2Re+Re)} - \frac{Gme^{-ReSy}}{Re(S^2-S)}$$

$$=y\mathrm{Gm}e^{-Rey} - \frac{Gre^{-RePry}}{Re(Pr^2-\mathrm{pr})}$$

When S=1 ,Pr=1, then we have

PI=y(Gr+Gm)e^{-Rey}

At y→ $\infty, u_0(y) = U \Rightarrow C_6 = U$

And y=0 , $u_0(y) = V \Rightarrow C_5 = V - U - (PI)_{y=0}$

Therefore , the solution $u_0(y)$ *are given by*

$$u_0 = U + (V-U)e^{-Rey} + \frac{Gr}{RePr(Pr-1)}(e^{-Rey} - e^{-Rey}) + \frac{Gm}{ReS(S-1)}(e^{-Rey} - e^{-ReSy}), \text{ for Pr} \neq 1, \text{S} \neq 1$$

$$u_0 = U + (V-U)e^{-Rey} + Grye^{-Rey} + \frac{Gm}{ReS(S-1)}(e^{-Rey} - e^{-ReSy}), for\ Pr = 1, S \neq 1$$

When $\varepsilon \neq 0$,substituting into the equation to and equating the coefficient of the following system of linear partial differential equations of first order are obtained :

$$\frac{\partial v_1}{\partial y} + \frac{\partial w_1}{\partial z} = 0$$

$$v_1 \frac{\partial u_0}{\partial y} - \frac{\partial u_1}{\partial z} = Gr\theta_1 + GmC_1 + \frac{1}{Re}\left(\frac{\partial^2 u_1}{\partial y^2} + \frac{\partial^2 u_1}{\partial z^2}\right),$$

$$\frac{\partial v_1}{\partial y} = -\frac{1}{Re^2}\frac{\partial p_1}{\partial z} + \frac{1}{Re}\left(\frac{\partial^2 v_1}{\partial y^2} + \frac{\partial^2 v_1}{\partial z^2}\right),$$

$$\frac{\partial w_1}{\partial y} = -\frac{1}{Re^2}\frac{\partial p_1}{\partial y} + \frac{1}{Re}(\frac{\partial^2 w_1}{\partial y^2} + \frac{\partial^2 w_1}{\partial z^2})$$

With the corresponding boundary conditions:

$$y = 0: u_1 = 0, v_1 = -cos\pi z, w_1 = 0, \theta_1 = 0, P_1 = 0, C_1 = 0$$

$$y \to \infty: u_1 \to 0, v_1 \to 0, w_1 \to 0, \theta_1 \to 0, P_1 \to 0, C_1 \to 0$$

The equations and govern the cross flow and equations like govern the main flow ,the temperature and the species concentration respectively.

1.6 CROSS FLOW SOLUTION:

In order to solve the equation and being independent of the main flow component u_1 and the temperature field , we assume that

$$v_1(y,z) = -\pi v_{11}(y)cos\pi z$$

$$w_1(y,z) = v''_{11}(y)sin\pi z$$

$$P_1(y,z) = Re^2 P_{11}(y)cos\pi z$$

The prime in v'_{11} denotes differentiation with respect to y. Equation and has been chosen so that the continuity equation is satisfied.

Substituting these equation to into the equation and ,we obtain the following ordinary differential equation for v_{11} and p_{11} .

v''_{11}+Re$v'_{11} - \pi^2 v_{11}$=$-\frac{Rep'_{11}}{\pi}$

$$v'''_{11} + Rev''_{11} - \pi^2 v'_{11} = -\pi ReP_{11}$$

With boundary conditions

$$\text{Y=0:} v_{11} = \frac{1}{\pi}, v'_{11} = 0$$

$$y \to \infty: v_{11} \to 0, v'_{11} \to 0$$

From , we get

$$\frac{ReP''_{11}}{\pi} = \pi Rep_{11} \Rightarrow p''_{11} = \pi^2 p_{11}$$

Let us take the solution of $p_{11}(y)$ is

$p_{11}(y) = p_{11}e^{-\pi y}$,where p_{11} is constant.

$$\Rightarrow D^2 + ReD - \pi^2)v_{11} = Rep_{11}e^{-\pi y}, D = \frac{d}{dy}$$

Let us assume the solution of v_{11} is a constant and

$v_{11}(y) = v_{11}e^{-my} + PI$, where v_{11} is a constant and

$PI = \frac{1}{D^2+ReD-\pi^2} Rep_{11}e^{-\pi y}$

$$=\frac{1}{\pi^2-Re\pi-\pi^2} Rep_{11}e^{-\pi y} = -\frac{p_{11}}{\pi}e^{-\pi y}$$

$$\therefore v_{11}(y) = v_{11}e^{-my} - \frac{p_{11}}{\pi}e^{-\pi y}$$

At $v_{11}(0) = \frac{1}{\pi}, v'_{11}(0) = 0$, we have

$\frac{1}{\pi}=v_{11} - \frac{p_{11}}{\pi}$ and $0 = -mv_{11} + p_{11}$

$$\Rightarrow \frac{1}{\pi} = v_{11} - \frac{mv_{11}}{\pi} = (1 - \frac{m}{\pi})v_{11}$$

$$\Rightarrow v_{11} = \frac{1}{\pi-m} \text{ and } p_{11} = mv_{11} = \frac{m}{\pi-m}$$

$$\therefore v_{11}(y) = \frac{1}{\pi - m}e^{-my} - \frac{m}{\pi(\pi - m)}e^{-\pi y}$$

Also $p_{11}(y) = \frac{m}{\pi-m}e^{-\pi y}$

We obtain the solution of $v_{1,}w_1$ and p_1 as:

$$v_1(y,z) = \frac{1}{m - \pi}(\pi e^{-my} - me^{-\pi y})cos\pi z$$

$$w_1(y,z) = \frac{1}{m - \pi}(e^{-my} - e^{-\pi y})sin\pi z$$

$$p_1(y,z) = \frac{mRe^2}{\pi - m}e^{-\pi y}cos\pi z$$

Where m is positive root of $m^2 - Rem - \pi^2 = 0$ $i,e, m = \frac{1}{2}[Re + \sqrt{Re^2 + 4\pi^2}].$

1.7 SOLUTION FOR MAIN FLOW ,TEMPERATURE AND MOLAR CONCENTRATION FIELDS :

To solve the equation ,we have to reduce these equations into ordinary differential equation under the following assumptions :

H$(y,z) = H_1(y)cos\pi z$, when H stands for u_1 or C_1.

On substituting the equation into the equation , we obtain the following ordinary differential equationType equation here.

$$u''_{11} + Reu'_{11} - \pi^2 u_{11} = -\frac{Re}{m-\pi}[\pi e^{-my} - me^{-\pi y}]u'_0 - ReGr\theta_{11} - ReGmC_{11}$$

$$\theta''_{11} + RePr\theta'_{11} - \pi^2\theta_{11} = -\frac{RePr}{m-r}[\pi e^{-my} - me^{-ry}]\theta'_0$$

$$=\frac{Re^2Pr^2}{m-r}[\pi e^{-my} - me^{-ry}]e^{-RePry}$$

$$=\frac{Re^2Pr^2}{m-r}[\pi e^{-my} - me^{-\pi y}]e^{-RePry}$$

$$c''_{11} + RePrC'_{11} - \pi^2 C_{11} = -\frac{ReS}{m-r}[\pi e^{-my} - me^{-\pi y}]C'_0$$

$$=\frac{Re^2S^2}{m-r}[\pi e^{-my} - me^{-\pi y}]e^{-ReSy}$$

$$=\frac{Re^2S^2}{m-r}[\pi e^{-(m+ReS)y} - me^{-(\pi+ReS)y}]$$

With boundary conditions :

$$y = 0: u_{11} = 0, \theta_{11} = 0, C_{11} = 0$$

$$y \to \infty: u_{11} \to 0, \theta_{11} \to 0, C_{11} \to 0$$

HEAT TRANSFER:

In the dynamics of viscous fluid one is not much interested to know all the details of the velocity and temperature fields but would certainly like to know quantity of heat exchange between the body and the fluid .since at the boundary the heat exchange between the fluid and the body is only due to condition , according to fourier 's law , we have

$$\bar{q}_w = -k\left(\frac{\partial \bar{T}}{\partial \bar{y}}\right)$$

Where $\bar{y}$ is the direction normal to surface of the body .with the help of the coefficient of heat transfer can be calculated in non – dimensional form ,which is generally known as nusselt number as follows:

$$\text{Nu}=\frac{-\bar{q}_w}{\rho v_0 c_p (T_w - T)_\alpha}$$

MASS TRANSFER :

The relation between species transfer by convection and the concentration boundary layer may be demonstrated by recongnizing that the molar flux associated with species transfer by diffusion ,according to Ficks law it has the from

$$\bar{q}_m = -D\left(\frac{\partial \bar{C}}{\partial \bar{Y}}\right)$$

1.8 DISCUSSION :

To be realistic during the course of numerical calculations of velocity , temperature, species concentration , skin – friction co efficient and both the fluctuating parts of heat mass transfer , the values of prandtl number (pr)are chosen for mercury (Pr=0.025),aie at 20°C (Pr=0.71). the values of Schmidt number (S) are considered in such a way that they represent the diffusing Chemical Species of most common interest in air . here two cases og general interest for

Grashoff number Gr >0 corresponding to cooling of the plate and Gr<0 corresponding to heating of the plate are considered.

The main flow velocity (u) have been shown in due to the cooling of the plate and heating of the plate respectively when $\Pr \neq 1, S \neq 1$.it is noticed that an increase in Re ,Pr and S leads to a decrease in u, where as the behaviour of u is reversed for the parameters V and Gm .the velocity of fluid layer decreases in magnitude for thicker diffusing species and substantial decrease is observed near the plate . increasing Re,Gm,V and Pr result in increase in magnitude and extent of velocity in case of heated plate . also that heavier diffusing foreign species increasing Schmidt number reduction in velocity level both in magnitude and extent .Comparison of velocity distribution curves shows that the curves fall gradually after attaining a maximum value near the plate or on the plate. Again substantial increase or decrease occurs in velocity distribution near the plate in case of heated plate . the fluid velocity u approaches zero for the large values of Re.

The fluid temperature is presented graphically against y for different values Re and Pr . it is ob severd that an increase in Re and Pr decreases the temperature field indicating that falls more rapidly for water in comparison to air .

1.9 CONCLUSION:

The above study brings out the following results of physical

interest :

1. Due to cooled plate the prandtl and Reynolds number has a retarding effect on the velocity of the flow field , where as the platr velocity and Gm accelerates u.
2. Due to heated plate , the Gm ,Pr,V, And Re has an accelerating effect on the velocity of the flow field .

3. Whether the plate is cooled or heated , the Schmidt number has a retarding effect on u .
4. Due to cooling of the plate, the values of u in the case of water and water at 4°C are much less in the case of ammonia and this behaviour is reversed to the heating of the plate.

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