AN ANALYTICAL STUDY OF DIFFERENT DIVISIBILITY TESTS ON BASE 2(BINARY), 9(OCTAL) AND 16(HEXADECIMAL)

Dissertation submitted to the Department of Mathematics in partial fulfillment of the requirements for the award of the degree of Master of Science in Mathematics

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Certificate

This is to certify thatCHARMIN RAHMAN bearing **Roll No MAT-27/23** and **Regd. No. MSSV-0023-101-001521**has prepared her dissertation entitled "**AN ANALYTICAL STUDY OF DIFFERENT DIVISIBILITY TESTS ON BINARY (BASE2), OCTAL (BASE8) AND HEXADECIMAL (BASE16)**" submitted to the Department of Mathematics, **MAHAPURUSHA SRIMANTA SANKARADEVA VISWAVIDYALAYA,** Nagaon, for fulfillment of MSc. degree, under guidance of me and neither the dissertation nor any part thereof has submitted to this or any other university for a research degree or diploma.

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DECLARATION

I, Charmin Rahman bearing the Roll No – MAT- 27/23, hereby declare that this dissertation entitled, "An analytical study of different divisibility tests on binary (base2),Octal (base8),and hexadecimal (base16)" was carried out by me under the supervision of my guide Dr. Maitrayee Chowdhury Ma'am, Assistant Professor, Department of Mathematics, Mahapurush Srimanta Sankardev Viswavidyalaya, Nagaon. The study and recommendation drawn are original and correct to the best of my knowledge.

Charmin Rahman

Date: M.Sc. 4^{th} semester

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ABSTRACT

This analytical study explores the divisibility tests within three fundamental number systems: binary (base2), Octal (base8), and hexadecimal (base16). These systems are widely used in computer science, digital electronics, and programming , but their mathematical properties – especially divisibility rules – are not commonly explored in depth. The study aims to understand how numbers behave under division in each base and to develop or explain rules that can quickly test for divisibility by certain numbers, similar to what is done in the decimal system.

In the binary system, for example, a number is divisible by 2 if it ends in 0. In octal and hexadecimal, the rules depend on grouping binary digits and understanding place values. The analytical study of divisibility test across the three systems highlight patterns, similarities, and differences. It also explain how such knowledge is useful in simplifying calculations in digital systems.

1. Introduction

Divisibility is a fundamental concept of mathematics. It determines whether one integer is divisible by another without leaving a remainder. A divisibility test is an efficient method to check this condition without performing full division. These tests play a crucial role in simplifying calculations identifying prime numbers, factoring integers.

Divisibility rules are well established in base 10, our common system. However, with the advancement of digital systems and computing alternative bases such as binary (base 2), Octal (base 8), and hexadecimal (base 16) have become increasingly important.

This analytical study focuses on exploring and forming divisibility tests in (base 2),(base 8) and (base 16). The concept of addition, subtraction and multiplication are basic but lead to more complicated concepts, like divisibility.

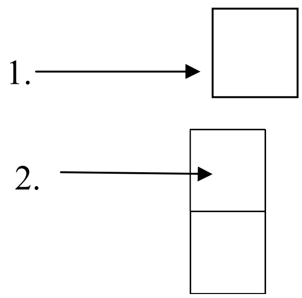
Recall that $Z = \{0, \pm1, \pm2, \pm3 \ldots\ldots\ldots\ldots\ldots..\}$

Divisibility is a statement about two integers a and b. Informally, an integer a divides b if a goes exactly into b.

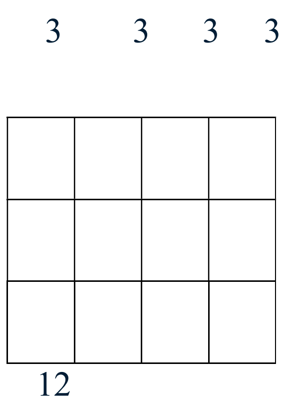
Geometric idea of Divisibility:

(Only for positive integers)

Represent numbers by squares, like so,



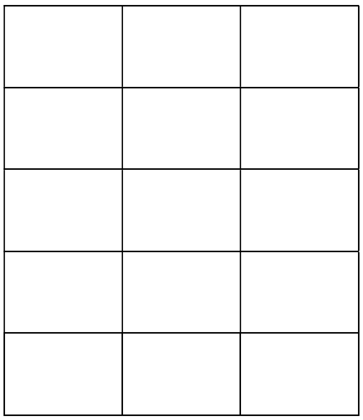
Eg.1: 12 is divisible by 3



3 goes into 12 four times

Eg.2: 15 is divisible by 5

5 5 5



15

5 goes into 15 three times

1.1 DEFINITION OF DIVISIBLITY:

Let a(≠0) and b be integers. We say that a divides b, denoted by a | b, if and only if there is an integer $k \in z$ such that b= k.a

There is no contradiction if we allow a=0 in the definition.

If a=0 in the definition, then we have that 0/b or b=k.0 for some integer k.

This only happens when b=0

Examples of divisibility rules:

1. Divisibility by 2: If the digit units place is even.

   eg.0, 2, 4, 6 or 8

2. Divisibility by 3: If the sum of the digit is divisible by 3

   eg. 315 ⟶ 3+1+5=9

3. Divisibility by 4: If the last 2 digit are divisible by 4

   eg. 644 ⟶ 44÷4=11

4. Divisibility by 5: If the last digit is 0 or 5

   eg.25, 60, 120

5. Divisibility by 6: If the number is divisible both 2 and 3.

   eg.48

6. Divisibility by 9: If the sum of the digits is divisible by 9

   eg. 324 ⟶ 3+2+4

1.2 Historical background of Divisibility Test:

The study of divisibility lies at the core of number theory and has evolved over centuries, influenced by diverse mathematical traditions. The development of divisibility tests reflects both practical and arithmetic needs and deeper theoretical insight in mathematics.

(i) Ancient Mathematics

The earliest evidence of arithmetic operations, including rudimentary forms of divisibility, dates back to **Babylonian and Egyptian** mathematics (2000–500 BCE). These civilizations used tables and repeated subtraction or multiplication to identify factors, though no formal divisibility tests were documented. Their work laid the groundwork for recognizing numerical patterns—an essential step towards formal divisibility rules.

(ii). **Greek Contributions**

The **ancient Greeks**, particularly **Euclid** (~300 BCE), made substantial contributions to the theory of numbers. In *Euclid's Elements*, Book VII outlines fundamental results concerning primes, common divisors, and the **Euclidean algorithm**, which is an early formal method relying on the concept of divisibility. Although not expressed in modern notation, these ideas formed the conceptual basis for modern divisibility tests.

(ii) Indian Mathematics

Significant progress in understanding number properties emerged in **classical Indian mathematics**:

- **Āryabhaṭa (476–550 CE)**, in his work *Āryabhaṭīya*, explored modular arithmetic and positional number systems, laying the foundation for divisibility in different bases.
- **Bhāskara II (1114–1185 CE)** elaborated on these ideas in *Līlāvatī* and *Bījagaṇita*, using congruences and introducing rules that

resemble divisibility checks.

These works demonstrated an advanced understanding of arithmetic operations and formed the early basis for algorithmic thinking in divisibility.

(iv). Islamic Mathematics

During the **Islamic Golden Age** (8th–13th centuries), scholars like **Al-Khwarizmi** and **Omar Khayyam** preserved and expanded upon Greek and Indian knowledge. They introduced systematic methods of calculation, some of which involved recognizing divisible numbers, especially in algebraic contexts. Their translations and commentaries helped transmit these ideas to medieval Europe.

(v). European Renaissance and Early Modern Period

In the 15th and 16th centuries, the revival of mathematics in Europe brought greater attention to arithmetic instruction. Mathematicians like **Luca Pacioli** and **Simon Stevin** compiled practical arithmetic rules, including divisibility tests for 2, 3, 5, 9, and 11 in base-10, for use in commerce and education. These rules, often heuristic, became standard in school curricula.

(vi). Formalization in Modern Number Theory

The formal structure of divisibility emerged prominently in the 18th and 19th centuries:

- **Carl Friedrich Gauss**, in his seminal work *DisquisitionesArithmeticae* (1801), introduced the formal framework of **modular arithmetic**, giving a rigorous basis to many divisibility rules.
- Concepts such as **congruences**, **residue classes**, and **divisibility criteria** were now grounded in abstract reasoning, moving beyond empirical rules to systematic theory.

(vii). Contemporary Applications and Base Systems

In modern times, divisibility rules have been extended to non-decimal bases such as:

- **Binary (base-2)** – crucial in digital computing and error detection.
- **Octal (base-8)** and **Hexadecimal (base-16)** – used in computer architecture, memory addressing, and programming.These adaptations of classical divisibility principles into different numeral systems reflect their ongoing relevance in mathematics, computer science, and engineering.

2. Literature Review

Tiebekabe in his work “New Divisibility Tests” focuses on the divisibility rule of 7 in the decimal base. He also focuses on the divisibility test or rule given by Nigerian youth Chika Ofili and demonstrates the rule given by Ofili. Also, the author tries for a new divisibility test of 3 and compares with various other standard tests that are already in use.

Chairux in his work “Chairux Algorithm for divisibility test” presents a novel idea named Chairux Algorithm for the divisibility test. It is based on an arbitrary integer and also has its base on the famous Bezout’s identity and the Euclidean algorithm. It provides a simple and interesting, also efficient approach to test the divisibility of numbers.

Cherniavsky and Mouftakhov’swork named “Zbikowski’s divisibility Criterion” is a simple, quick and efficient method where one can obtain a criterion that determines whether one integer is divisible by another. It is a method that is easier than using long division or Pascal’s test of divisibility and it can be explained to students at any level.

Abdulbaqi et al., in his research paper “General rules of evaluating binary number divisibility on primes numbers” defines a new rule to find the divisibility of a binary numbers by a prime number greater than 2. The rule used here is to separate the binary number into blocks of bits. After this each block is then processed separately in a special procedure to find the possible divisibility on the prime number. This rule is tested with prime numbers 3, 5 and 7.

3. Preliminaries

Definition 3.1: Binary (base 2): The binary number system is a system of representing numbers using only two digits 0 and 1. It is also known as the base-2 number system, as it uses only two digits as its base. It is used in computer systems and electronic devices to represent data and instructions, as the two digits can be easily represented by the presence or absence of an electrical signal.

Example:

$$(10101)_2 = 1\times 2^4 + 0\times 2^3 + 1\times 2^2 + 0\times 2^1 + 1\times 2^0$$

$$= 1\times 16 + 0 + 4 + 0 + 1$$
$$= 21$$

Definition 3.2: Octal (base 8): Octal number system has eight digits- 0, 1, 2, 3, 4, 5, 6 and 7. Octal number system is also a positional value system with where each digit has its value expressed in powers of 8, as shown here-

8^5	8^4	8^3	8^2	8^1	8^0

Example: $(52)_8 = 5\times 8^1 + 2\times 8^0$

= 42 + 2

=42

Octal number system table:

We use only 3 bits to represent octal numbers. Each group will have a distinct value between 000 and 111.

Octal Digit	Decimal value	3- bit Binary number
0	0	000
1	1	001
2	2	010
3	3	011
4	4	100
5	5	101
6	6	110
7	7	111

- As its base is $8 = 2^3$, every symbol of this symbol can be represented by its three bit binary equivalent.
- The binary digits $(8 = 2^3)$ are efficient to represent any octal digit.

- Ex: $(100101001010)_2$

$= (4512)_8$

Usage of Octal (base 8):

- Compact representation of the binary numbers.
- Reduces the chances of errors.
- Easily conversion in Binary.

Definition 3.3: Hexadecimal (base 16): The number system with base 16 that uses ten numbers from 0-9 and six letters up to A to F is called Hexadecimal number system.

Here, A=10, B=11, C=12, D=13, E=14, F=15

Example: $(A03)_{16} = A\times16^2 + 0\times16^1 + 3\times16^0$

$= 10\times256 + 0 + 3$

$= 2563$

Usage of Hexadecimal (Base16): The hexadecimal number (base 16), employing sixteen distinct symbols 0-9 and A-F, plays a significant role in both theoretical and applied mathematics,especially in areas intersecting with number theory, computer science, and digital electronics. Its importance stems from its compact representation and strong compatibility with binary (base2) systems.

4. Divisibility tests in Binary, Octal and Hexadecimal-an overview

4.1 Divisibility of binary numbers by 2:

First, we observe that if the last digit of a number in binary base is 0 then, when we convert it into decimal system the first value would be $0 \times 2^0 = 0$ and then further towards the left whatever be the digits we would eventually get that as an even number.

$(101)_2$ in decimal system would be,

$$1 \times 2^0 + 0 \times 2^1 + 1 \times 2^2 = 1 + 2 + 4 = 7$$

For example:$(10)_2$ in decimal system is

$$0 \times 2^0 + 1 \times 2^1 = 0 + 2 = 2$$

Again, suppose we take, $(110)_2$ then in decimal system it would be

$$0 \times 2^0 + 1 \times 2^1 + 1 \times 2^2 = 0 + 2 + 4 = 6$$

And if we the the last digit not equal to 0 then we will get an odd number.

Suppose, we take,

$(11)_2$ in decimal system would be,

$$1 \times 2^0 + 1 \times 2^1 = 1 + 2 = 3$$

So, whenever we see that the last digit of a number in binary system is 0 then the number is divisible by 2.

4.2 Divisibility by 4:

In case of divisibility by 4 if we have the last two digits of the number in binary system as 00, then we observe that in decimal system we would get

$$0 \times 2^0 + 0 \times 2^1 + 0 / 1 \times 2^2$$

This finally would be a number that is divisible by 4.

Example: $(1100)_2$ is base 10 to be

$$0 \times 2^0 + 0 \times 2^1 + 1 \times 2^2 + 1 \times 2^3 = 0 + 0 + 4 + 8 = 12$$

Let us take another example, say, $(101100)_2$ would give us,

$$0 \times 2^0 + 0 \times 2^1 + 1 \times 2^2 + 1 \times 2^3 + 0 \times 2^4 + 1 \times 2^5$$
$$= 0 + 0 + 4 + 8 + 0 + 32$$
$$= 44$$

That is divisible by 4

So, whenever the last two digits of a number in binary system is 0 then the number is divisible by 4

4.3 Divisibility by 8:

For divisibility by 8 in case of a number in binary system it is important to note that if the last three digits of the number is 0 then the number in decimal system would be,

$$0 \times 2^0 + 0 \times 2^1 + 0 \times 2^2 + 1\ (or\ 0) \times 2^3 + 1(or\ 0) \times 2^4 + \cdots$$

This number would be a multiple of 8.

For example: Suppose we take,

$(101000)_2$ this number in decimal system would be

$$0 \times 2^0 + 0 \times 2^1 + 0 \times 2^2 + 1 \times 2^3 + 0 \times 2^4 + 1 \times 2^5$$

$$= 0 + 0 + 0 + 8 + 0 + 32$$

$= 40$, which is divisible by 8

4.4 Divisibility of a number in binary system by 3:

For a number in binary system to be divisible by 3 the method of alternating sums is used where the weights +1 and -1 are alternating multiplied with the digits of the number in binary system from the extreme right. After that the total of all this product is added and if the total is equal to zero then the number is divisible by 3 and if the total is not equal to zero then the number is not divisible by 3.

For example: Let us check whether $(1101)_2$ is divisible by 3 or not.

For this according to the above mention rule we calculate in the following way,

$$1 \times 1 + 0 \times (-1) + 1 \times 1 + 1 \times (-1)$$
$$= 1 + 0 + 1 - 1$$
$$= 1$$

Since 1 is the divisible by 3

So therefore, $(1101)_2$ is not divisible by 3

We look into another example,

Check whether $(1002)_2$ is divisible by 3 or not. For this in the similar way as done for the previous example we calculate the following,

$$1 \times 1 + 0 \times (-1) + 0 \times 1 + 1 \times (-1)$$
$$= 1 + 0 + 0 - 1$$
$$= 0$$

Since it is zero, therefore $(1001)_2$ is divisible by base 3.

4.5 Divisibility in octal base by 2:

For divisibility of a number in octal base to be divisible by 2 we observe that whenever the last digit in octal base is even i.e 0, 2, 4 or 6 then after converting it to base 10 we will get the number i.e. even and is a multiple of 8.

But in case the last digit in the octal base is odd then after converting it to base 10. We will finally get and odd number which is obviously not divisible by 8.

Check whether $(231)_8$ is divisible is divisible by 2?

Here, the last digit is 1, which is odd therefore by the above rule $(231)_8$ is not divisible by 2.

Detailed conversion of $(231)_8$ to decimal,

$$(231)_8 = 1\times 8^0 + 3\times 8^1 + 2\times 8^2$$

=1+24+128

=153, which is not divisible by 8

Furthermore, let us take another example say

$(352)_8$ And check whether it is divisible by 2

Since, the last digit is 2 so by the above rule $(352)_8$ should be divisible by 8.

Detailed conversion of,

$(352)_8 = 2 \times 8^0 + 5 \times 8^1 + 3 \times 8^2$

=2+40+192

=234, which is divisible by 8.

4.6 Divisibility in octal base by 4:

For divisibility of a number in octal base by 4 we observe that if the last two digits in base 8 of the number is divisible by 4 then the no is divisible by 4 in base 10.

In $(234)_8$ the last two digits are 3 and 4. Also,

$(34)_8 = 4 \times 8^0 + 3 \times 8^1$

$= 4+24= (28)_{10}$

Here 28 is divisible by 4.

So, $(234)_8$ is divisible by 4.

Detailed conversion: $(234)_8$

$= 4\times 8^0 + 3\times 8^1 + 2\times 8^2$

$= 4+24+128$

$= (156)_{10}$

Which is divisible by 4.

Furthermore, let us check for divisibility of $(321)_8$ by 4.

Here the last two digits are 2, 1

Also, 21 is not divisible by 4.

Hence according to the above rule $(321)_8$ is not divisibility by 4 in base 10.

Detailed conversion,

$(321)_8 = 1\times 8^0 + 2\times 8^1 + 3\times 8^2$

$=1+16+192$

= 209, which is not divisible by 4.

4.7 Divisibility in Octal base by 3:

For divisibility of an octal base number by 3 it is observe that if the sum of the digits is divisible by 3 then the number in octal base is divisible by 3.

For example, let us check whether $(152)_8$

$$2\times 8^0 + 5\times 8^1 + 1\times 8^2$$

$= 2+40+64$

$=106$, which is divisible by 3.

Furthermore, let us take another number is octal base say, $(255)_8$

Here the sum of the digits is 2+5+5=12 which is divisible by 3.

Hence, the number $(225)_8$ is divisible by 3 according to the above rule.

Detailed conversion,

$$(225)_8 = 5\times 8^0 + 5\times 8^1 + 2\times 8^2 \backslash$$

$= 5+40+128$

= 173, which is not divisible by 3.

4.8 Divisibility of hexadecimal numbers by 2:

In case of hexadecimal numbers, we have the digits 0-9 and letters A-F here even numbers are0, 2, 4, 6, 8, A, C, E in case of divisibility by 2 the last digit in hexadecimal base should be even for if it is an odd number. It is evident that after conversion in base 10 the number would be odd one.

For example:

Suppose we take $(123)_{16}$ where the last digits in an odd number. According to the above rule this number will not be divisible by 2.

Detailed conversion,

$$3\times16^{0}+2\times16^{1}+1\times16^{2}$$

= 3 + 32 +256

= 291, which is not divisible by 2.

But, let us above the number $(214)_{16}$

Here, the last digit is an even number so according to the divisibility rule. This number should be divisible by 2.

Detailed conversion,

$(214)_{16} = 4\times16^0 + 1\times16^1 + 2\times16^2$

$= 4 + 16 + 512$

$= 532$, which is divisible by 2.

4.9 Divisibility of Hexadecimal base by 4:

For a number in hexadecimal base to be divisible by 4 it is observed that the last digit of the number should be divisible by 4 i.e., 0, 4, 8 or C. Also, if the last two digits are 0 then the number is divisible by 4.

For example: suppose we consider the hexadecimal number $(318)_{16}$. Here the last digit is 8 that is divisible by 4. So, according to the above law the number $(318)_{16}$ is divisible by 4. Again, we observe the number $(318)_{16}$ The last digit here is 2 which is not divisible by 4

Detailed conversion of this number would be

$2 \times 16^0 + 1 \times 16^1 + 3 \times 16^2$

$= 2 + 16 + 768$

$= 786$, which is not divisible by 4

4.10 Divisibility of Hexadecimal base by 8:

In case of a hexadecimal based number to be divisible by 8 it is observed that the last digit should be wither 0 or 8. Otherwise it is also divisible if the last three digits of the number is0.Check whether,

$(326)_{16}$ is divisible by 8

Here, the last digit is 6 which is not divisible by 8 and so according to the above-mentioned rule the number is not divisible by 8.

Detailed conversion of 326_{16} would be

$6 \times 16^0 + 2 \times 16^1 + 3 \times 16^2 = 6 + 32 + 768$

$= 806$, which is not divisible by 8.

But let us now consider another example say $(348)_{16}$

In detailed conversion the number would be,

$8 \times 16^0 + 4 \times 16^1 + 3 \times 16^2$

$= 8 + 64 + 768$

$= 840$, which is divisible by 8.

5. Conclusion and Future scope

Divisibility tests are straightforward mathematical rules that show whether one number can be divided by another without leaving a remainder. These tests are helpful in arithmetic, number theory, and real-world applications. They make calculations easier, boost mental math skills, and deepen our understanding of math concepts. Their usefulness ranges from classroom learning to solving everyday problems. One key way divisibility tests are helpful is by simplifying calculations. When working with large numbers, using long division to check for divisibility can take a lot of time. Instead, applying rules like "a number is divisible by 3 if the sum of its digits is divisible by 3" gives the answer quickly. This saves time and lowers the chance of making mistakes, especially during exams or when under pressure. Divisibility rules are also important for simplifying fractions. To reduce a fraction to its simplest form, we need to find common factors of the numerator and denominator. Divisibility tests help us identify these common factors quickly. For example, knowing that both 84 and 126 are divisible by 3 and 7 makes it easy to reduce the fraction 84/126 without guessing divisions. In problem-solving and number theory, divisibility tests are essential. Many problems involving factors, multiples, prime numbers, and least common multiples (LCM) depend on understanding divisibility. For instance, determining if a number is prime requires checking its divisibility by smaller numbers. Efficiently testing for divisibility by 2, 3, 5, 7, or 11 lets us quickly rule out non-prime numbers. These tests also improve mental math skills. With practice, students learn to use the rules in their minds without writing everything down. This boosts their number sense and confidence in tackling complex arithmetic. Their mental agility grows as they spot patterns in numbers and use logical shortcuts instead of simple calculations. In algebra and higher-level math, divisibility concepts connect to subjects like modular arithmetic, which is important in cryptography, computer science, and coding theory. A solid grasp of basic divisibility lays the groundwork for these advanced topics. Beyond pure math, divisibility tests have real-life applications. For instance, when dividing items evenly among groups, checking divisibility helps ensure fairness. In finance, inventory, or packaging, businesses often need to divide amounts efficiently—divisibility rules help them make quick decisions. Programmers and engineers also use these rules when creating algorithms to optimize processes involving division or grouping. In summary, divisibility tests are not just tricks; they are valuable tools that improve efficiency, understanding, and logical thinking. From simplifying arithmetic to solving complex issues and making practical choices, these tests aid both learners and professionals. Mastering them early in education establishes a strong mathematical foundation that benefits lifelong learning and problem-solving.

The main aim of the divisibility tests is to reduce the amount of time it takes by actually dividing two numbers. It is with this thought in mind and also the inquisitiveness of the combinatorial part of the human mind that has led to the discovery of the tests. As a result of which it has definitely made our work easier to a great extent. It is an innovative area of combinatorial mathematics where we can study the different ways of divisibility and try to find out more ways of how to do it.

This analytical study has explored and compared the divisibility rules in three key positional number systems: binary (base 2), octal (base 8), and hexadecimal (base 16). Each base has its own unique structure and application, and understanding their divisibility rules not only enhances number theory knowledge but also has practical value in digital electronics, computer science, and coding theory. Binary, being the foundation of all digital computation, offers the simplest rules. Octal and hexadecimal, used for compact representations in computing, demonstrate interesting patterns that extend or modify decimal-based divisibility concepts.

Through this study, we found that divisibility tests in these non-decimal bases often rely on pattern recognition, positional value, and modular arithmetic. These rules differ from traditional decimal rules and can be generalized using algebraic or base-conversion methods.

Future Scope

Algorithm Development: The study can be extended to create efficient algorithms for automated divisibility checking in different number bases. Generalization to Other Bases: Research can be broadened to include other bases (e.g., base 3, 5, 12) to identify universal patterns or formulate generalized divisibility rules.

Application in Cryptography and Coding: Exploring the role of divisibility in error-checking, data compression, and cryptographic key design.

Integration with Machine Learning: Using pattern recognition to automatically derive or predict divisibility rules for large bases.

Educational Tools: Development of interactive software for teaching divisibility concepts across different number systems.

Divisibility tests have been crucial tools in arithmetic and number theory. They help people quickly determine if one number can be evenly divided by another without doing full division. Traditionally taught in the decimal system, divisibility rules simplify mental math, assist in reducing fractions, and aid in understanding number properties. With technology advancing and the growing reliance on digital systems, divisibility tests are now relevant beyond just decimal arithmetic. This includes applications in binary, octal, and hexadecimal systems. To fully grasp the future role of divisibility tests, we must look at their importance in education and mental math as well as in programming, computer science, cryptography, data systems, and emerging technologies.

1. **In the Decimal System (Base-10)** In the decimal system, divisibility tests are basic tools for education and computation. As math education evolves, there is more focus on numeracy and logical thinking. Divisibility tests offer a clear, pattern-based method that helps students develop foundational number sense. In future classrooms, with more artificial intelligence and digital learning platforms, automated math tutors might use these rules to explain problem-solving steps. This reinforces the importance of understanding mathematical patterns. Divisibility tests will remain crucial in this context, helping learners find shortcuts, check calculations, and improve algebraic thinking. In practical applications like financial computing, statistical data processing, and

automated testing, divisibility rules help improve algorithms that manage large datasets. For instance, determining when values are evenly distributed or partitioned often relies on divisibility checks.

2. **In the Binary System (Base-2)** The binary system is the backbone of modern computing. Every operation in a digital device, from simple calculations to complex data processing, happens in binary. In binary, a number is divisible by 2 if it ends with 0. This rule is very useful in bitwise operations where binary shifts and logic operations play a major role. As computing evolves toward quantum and neuromorphic systems, binary arithmetic remains important in classical computing. Divisibility by powers of 2 is key to optimizing performance in data storage, compression algorithms, and encryption systems. For example: - A number is divisible by 4 if its last two binary digits are 00. - A number is divisible by 8 if its last three digits are 000. These patterns are easy to spot in binary and are commonly used in system design, where data alignment and memory addressing depend on powers of 2. Divisibility tests in binary therefore support efficient designs, network protocols, and compiler optimization.

3. **In the Octal System (Base-8)** The octal system is often used in computer systems and embedded programming because of its close ties to

binary. Since one octal digit corresponds to exactly three binary digits, it makes reading and representing binary-coded data simpler. Divisibility in the octal system can aid with permission settings in Unix-based systems, file mode representations, and digital circuit configurations. For example, when dealing with memory mapping or device registers, knowing whether certain values are divisible by 2, 4, or 8 can improve design efficiency and debugging. As IoT and microcontroller systems become more widespread, understanding and applying octal-based divisibility rules will help developers optimize resource use and hardware communication.

4. **In the Hexadecimal System (Base-16)**: The hexadecimal system is commonly used in programming, digital color representation, cryptography, and memory addressing. Since one hex digit represents four binary digits, it provides a compact way to illustrate large binary numbers. Divisibility tests in hexadecimal are not as straightforward as in decimal, but patterns still exist. For example: - A hexadecimal number ending in 0 is divisible by 16. - Numbers ending in even hex digits (0, 2, 4, 6, 8, A, C, E) are divisible by 2. More complex algorithms can check for divisibility by numbers like 3 or 5 in hexadecimal, often involving modular arithmetic. In the future, as software applications become more

automated and optimized, quick checks for divisibility in hexadecimal could improve error-checking algorithms, hashing techniques, memory allocation, and data encryption. Cryptographic systems rely heavily on modular arithmetic and divisibility. Efficient checks in various number systems can enhance cryptographic key generation, checksum validation, and other verification processes. In cybersecurity, these patterns will continue to shape how data is encoded, validated, and protected.

5. **Educational and Research Implications**: With the rise of digital learning, teaching across different number systems will become more common. Learning tools that explain divisibility in base-2, base-8, and base-16 alongside base-10 will help students understand how different systems work and interact. This prepares them for careers in data science, software engineering, and robotics. Furthermore, ongoing research in mathematics looks at divisibility in abstract algebra, modular systems, and ring theory, all of which have applications in machine learning and advanced computational frameworks. Divisibility remains a key concept in algorithm development, especially when creating efficient, scalable, and reliable systems.

6. **Divisibility in AI and Automation**: As artificial intelligence systems advance, they rely on efficient data management. AI algorithms often need to divide data into segments or batches, and divisibility checks are key to figuring out how to split, process, and reassemble the data. For instance: - Training data might be divided into groups that are evenly divisible by a batch size. Hardware accelerators like GPUs and TPUs perform better when data is divisible by specific sizes. In automated testing and simulation, divisibility tests help set boundaries and cycles, improving simulation accuracy and reducing unnecessary computation. Conclusion Divisibility tests, while rooted in basic arithmetic, have a significant and growing role across modern and future technologies. In the decimal system, they support educational growth, mental math, and arithmetic efficiency. In binary, octal, and hexadecimal systems, they underpin computer architecture, digital communication, data security, and software optimization. As our world becomes more data-driven and computationally intensive, understanding and applying divisibility tests across number systems will keep becoming more important. Whether in programming, education, cryptography, AI, or advanced math, divisibility is more than just a math trick; it's a vital part of the digital age.

6. References

The books we followed for our work are listed below:

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