A STUDY

ON

THE HISTORY OF MATHEMATICS

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Certificate

This is to certify that **NAZIFA TASNIM** bearing Roll No. MAT-01/23 and **Regd No. MSSV-0023-101-001540** has prepared her dissertation entitled "**A STUDY ON THE HISTORY OF MATHEMATICS"** submitted to the department of Mathematics, Mahapurusha Srimanta Sankaradeva Viswavidyalaya (MSSV) Nagaon, for fulfilment of M.Sc. degree in Mathematics under the guidance of me neither the dissertation nor any part thereof has submitted to this or any other university for a research degree or diploma.

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DECLARATION

I hereby declare that this dissertation “A Study on the History of mathematics” submitted to MAHAPURUSHA SRIMANTA SANKARADEVA VISWAVIDYALAYA the award of the degree of Master of Science and Mathematics. This is my original work and has not been submitted previously by me or any other person for any other degree or diploma at this or any other institution.

This work has been carried out under the guidance of DR. MIRA DAS, Assistant professor, department of mathematics of MSSV. I further declare the sources of information and data used in the preparation of this dissertation have been duly acknowledge.

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ABSTRACT

This dissertation explores the transformative journey of the history of mathematics from ancient of civilizations to the period of modern centuries. It helps to thought how mathematics is originated in early societies such as Mesopotamia, Egypt, India, China and Greece where it was used in agriculture, trade, astronomy and architecture.

It developed the mathematical thought through the historical periods and including the Islamic golden age, the European Renaissance, Scientific revolution and the 17^{th} to 21^{st} centuries includes the development of calculus, geometry, logic, abstraction, computation and interdisciplinary applications.

Analysing the cultural, philosophical and scientific context in which the mathematics were developed. In this study we know the how human understanding of the natural world and also situated the foundation for modern science and technology.

CONTENT

LITERATURE REVIEW

The purpose of this thesis of The History of Mathematics is explore the development if mathematical ideas, concepts and techniques over time and across the different civilizations. It aims to the understand the origin of development of mathematics, cultural contribution, analyse the relationship between mathematics and society and growth of mathematical thought. The scope for the thesis it includes chronological, geographical, thematic and educational and cultural scope of the history of mathematics.

We discuss the historical periods and their contributions to mathematics such as ancient Mesopotamia, Egypt, Greek and the Islamic golden age and the European renaissance and we also discuss about the contributions of the figures such as Euclid, Archimedes, Fibonacci, newton and Euler.

We analyse the themes and concept of this thesis such as the development of number system, geometry, algebra, calculus and probability. Also this thesis explores the revolutionary changes in mathematics during the 17^{th} to 19^{th} centuries including the development of calculus, geometry and formal logic. It also examines the 20^{th} and 21^{st} centuries, marked by an abstraction, computation and interdisciplinary applications.

Examples of relevant literature: A History of Mathematics, by Boyer, Carl B., and Uta C. Merzbach a history of mathematics 2^{nd} ed., John Wiley & sons, 1991.

Krtz, Victor J. A history of mathematics: An introduction. 3^{rd} edition, Pearson Education, 2009. and, an approaches the history of mathematics by Far east journal.

CHAPTER-1

THE BASIC CONCEPT OF THE HISTORY OF MATHEMATICS

1.1. INTRODUCTION

The history of mathematics is a rich interdisciplinary field that explores the origins, development, and distribution of mathematical ideas throughout human history. This literature review surveys foundational and contemporary works that history of the evolution of mathematics, highlighting key civilizations, milestones, and thematic approaches to the discipline.

The origin of the Mathematics can be traced back to ancient times. The first evidence of mathematical activity dates back to the period between 35,000 and 25,000 BC in Africa. The earliest known mathematical texts dates back to the period 1900 to 1600 BC in the old Babylonian period of Mesopotamia.

However, they also contain some article problems that require These texts are mostly concerned with solving particle problems, such as how to divide an inheritance the use of simple algebra important and ubiquitous disciplines in the world. In the modern era mathematics is used in The study of mathematics has since then evolve and expanded to become one of the most a wide variety of fields, from physics and Engineering to finance, economics and the development of mathematical models, numerical mathematics and ICT related mathematics.

1.2. FOUNDATIONAL TEXTS AND AUTHOR

(i) Morris Kline:

Mathematical Thought from Ancient to Modern Times (1972).

Kline's work is one of the most comprehensive and concepts of single-volume histories of mathematics. It covers mathematical developments from Babylonian and Egyptian mathematics to the 20th century, emphasizing the interplay between mathematics and other fields such as philosophy, physics, and art.

(ii) Carl B. Boyer:

A History of Mathematics (1968, with revisions by Uta Merzbach).

This textbook is often used in university courses and provides a chronological account of mathematical ideas. Boyer's history is specially strong on the evolution of calculus and the contributions of European mathematicians during the Renaissance and awareness.

(iii) E.T. Bell:

Men of Mathematics (1937)

Although dated and more biographical than analytical, Bell's work provides engaging stories of well-known mathematicians. His romanticized style has inspired many readers, though it is often analyzed for historical mistake and gender bias.

CHAPTER-2

THE BIRTH OF MATHEMATICS IN ANCIENT CIVILIZATIONS

2.1. Introduction

Origin of mathematics is followed back to the earliest civilizations is need for counting, measuring and organizing the world is situated to the foundation for this essential disciplines. The ancient of civilizations of Mesopotamia, Egypt, the Indus Valley and China were among the first developed the mathematical concepts.

2.2: Ancient Egyptian Mathematics

Ancient of Egyptian mathematics is regarded as a basis of the history of mathematics. The ancient Egyptian mathematics is contained in the Rhind Mathematics papyrus, which from 1650 BC. This papyrus makes references to mathematical topics such as Fractions, Pythagorean triples and the geometry of triangles, Circles and Rectangles.

The Egyptians had a refined understanding of arithmatic, algebra and geometry. They were able to quickly solve complicated mathematical problems by calculating the second and third equation, as well as calculating the volume of objects like Pyramids and cylinders.

The Egyptian also made great step in astronomical calculations. By thoroughly keeping records of the movement of the stars, they developed the first accurate calendar with a 365 day cycle. This allowed them to plan agricultural activities more precisely than their predecessors.

Ancient Egyptian mathematics is credited with lying the foundation for many of the mathematical concepts we use today. It is an essential part of the history of mathematics and important source of inspiration for modern mathematicians.

2.2.1. Sources of Egyptian mathematics

Current understanding of ancient of Egyptian mathematics is block by the lack of available sources. The sources that do exists include the following texts:

. The Moscow Mathematical Papyrus.

. The Egyptian Mathematical Leather Roll.

. The Lahun Mathematical Papyri.

. The Berlin Papyrus 669,written around 1800 BC.

. The Akhmim Wooden Tablet.

. The Reisner Papyrus, dated to the early Twelfth dynasty of Egypt and found in Nag el –Deir, the ancient town of thinis.

. The Rhind Mathematical Papyrus (RMP),dated from the second intermediate period(c.1650 BC) ,but its author, Ahmes, identifies it as a copy of a now lost Middle Kingdom papyrus. The RMP is the largest mathematical texts.

From the New Kingdom there are handful of mathematical texts and inscriptions related to computations:

. The papyrus Anastasi I,a literary texts written as a letter written by a scribe named Hori and addressed to a scribe named Amenemope.A segment of the letter describes several mathematical problems.

. Ostracon Senmut 153, a text written in hieratic.

. Ostracon Turin 57170, texts written in hieratic.

. Ostraca from Deir el-Medina contain computations. Ostracon IFAO 1206 for instance shows the calculation of volimes, presumbly related to the quarrying of a tomb.

2.2.2. Algebra and Geometry

The ancient Egyptians were the first civilization to develop and solve the second –degree (quadratic) equations. This information is found in the Berlin Papyrus fragment. Additionally, the Egyptians solve first –degree algebraic equations found in Rhind Mathematical Papyrus.

There are only a limited number of problems from ancient Egypt that concern geometry. Geometric problems appear in both the Moscow Mathematical Papyrus (MMP) and in the Rhind Mathematical Papyrus (RMP). The examples show that the Ancient Egyptians know that how to compute areas of several geometric shapes and the volumes of cylinders and pyramids.

2.3. Ancient Babylonian Mathematics

The second great civilization in the history of mathematics is Babylonian origin. Babylonian mathematicians lived more than four millennia ago and their influence extended to both Egypt and Greece.

Babylonian mathematicians contributed to study of Astronomy, particularly in developing elegant techniques to calculate and predict the movement of orbits of the sun, moon and planets. Their mathematics was rooted in the concepts of a number base system of the number 60.

For example: 60 seconds in a minute and 60 minutes in an hour.

Babylonian mathematicians invented a more primitive form of algebra. They were also able to solve polygonal area problems and could calculate square and cube roots, as well as calculate the number [π].

An another contribution of Babylonian mathematics was use of the fractions. While we are familiar with the notation of fraction, today, the Babylonian was among the first to understand how to handle them with equally. Many of the techniques and methods used by the Babylonian are still used in modern mathematics.

2.3.1. Origin of Babylonian mathematics

Babylonian mathematics is a range of numeric and more advanced mathematical practices in the ancient near East, written in cuneiform script. Study has the historical focus on the first Babylonian rule old Babylonian period in the early second millennium BC due to the wealth of data available. There has a debate over the earliest appearance of Babylonian mathematics, with historians suggesting a range of dates between the 5-th and 3-rd millennia BC. Babylonian mathematics was primarily written on clay tablets in cuneiform script in the Akkadian or Sumerian languages.

Babylonian mathematics is perhaps an unhelpful term since the earliest suggested origin date to the use of accounting devices, such as bullae and tokens, in the 5-th millennium BC.

2.3.2. Babylonian numerals

The Babylonian systems of mathematics was a sexagesimal (base 60) numeral system From this we derive the modern –day usage of 60 seconds in a minute, 60 minutes in an hour and 360 degrees in a circle. The Babylonians were able to make great advances in mathematics for two reasons. Firstly, the number 60 is a superior highly composite number, having factors of 1,2,3,4,5,6,10,12,15,20,30,60 (including those that are themselves composite), facilitating calculations with fractions. Additionally, unlike the Egyptians and Romans, the Babylonian's had a true place value system, where digits wriiten in the left column represented larger values (much as, in our base ten system,

$734 = 7\times1000+3\times10+4\times1$).

2.3.3. Old Babylonian mathematics (2000-1600 BC)

Arithmetic

The Babylonians used pre-calculated tables to assist with arithmetic, including multiplication tables, tables of reciprocals and tables of square. Their multiplication tables were not the 60×60 tables that one might expect by analogy to decimal multiplication tables. Instead, they kept only tables for multiplication by certain Principal numbers. To calculate other products, they would split one of the numbers to be multiplied into a sum of principal number.

Algebra

As well as arithmetical calculations, Babylonian mathematicians also developed algebraic methods of solving equations.

To solve a quadratic equation the Babylonian's essentially used the standard quadratic formula. They considered quadratic equations of the form:

$$x^2 + bx = c$$

Where b and c were not necessarily integers, but c was always positive. They knew that a solutions to this form of equation is :

$$x = -\frac{b}{2} + \sqrt{\left(\frac{b}{2}\right)^2 + c}$$

And they found square roots efficiently using division and averaging. Problem of this type include finding the dimensions of a rectangle given its area and the amount by which the length exceeds the width.

Tables of values of $n^3 + n^2$ were used to solve certain cubic equations. For example, Consider the equation:

$$ax^3 + \mathrm{b}x^2 = c$$

Multiplying the equation by a^2 and dividing by b^3 gives:

$$\left(\frac{ax}{b}\right)^3 + \left(\frac{ax}{b}\right)^2 = \left(\frac{ca^2}{b^3}\right)$$

Substituting $y = \frac{ax}{b}$ gives

$$y^3 + y^2 = \frac{ca^2}{b^3}$$

Which now to solved by looking up the $n^3 + n^2$ table to find the value of closest to the right hand side. The Babylonian's accomplished this without algebraic notation, showing a remarkable depth of understanding. However, they did not have a method for solving the general cubic equation.

Plimpton 322

The Plimpton 322 tablet contains a list Pythagorean triples, that is, integers(a,b,c) such that $a^2 + b^2 = c^2$. The triples are too many and too large to have been obtained by brute force.

Geometry

Babylonians knew the common rules for measuring volumes and areas. They measured the circumference of a circle as three times the diameter and the area as one-twelfth the square of the circumference, which would be correct if π is estimated as 3.They were aware that this was an approximation and old Babylonian mathematical tablet excavated near Susa in 1936 (dated to between the 19-th and 17-th centuries BC) gives a better approximation of π as $\frac{25}{8} = 3.125$ about 0.5 per cent below the exact value. The volume of a cylinder was taken as the product of the base and the height, however, the volume of the frustum of a cone or a square pyramid was incorrectly taken as the product of the height and half the sum of the bases. The Pythagorean rule was also known to the Babylonians.

The Babylonian mile, was a measure of distance equal to about 11.3km (or about seven modern miles).This measurement for distances eventually was converted to a time-mile, used for measuring the travel of the sun, therefore representing time.

The Babylonian astronomers kept detailed records of the rising and setting of stars, the motion of the planets, and the solar and lunar eclipses, all of which required familiarity with angular distances measured on the celestial sphere.

They also used a form of Fourier analysis to computer an ephemeris (table of astronomical positions), which was discovered in the 1950's by Otto Neugebauer. To make calculations of the movements or celestial bodies, the Babylonian used basic arithmetic and a coordinate system based on the ecliptic, the part of the heavens that the sum and planets travel through.

Tablets kept in the British Museum provide evidence that the Babylonian's even went so far as to have a concept of objects in an abstract mathematical space. The tablets date from between 350 and 50 BC, revealing that the Babylonian's understood and used geometry even earlier than previously thought. The Babylonian's used a method for estimating the area under a curve by drawing a trapezoid underneath, a technique previously believed to have originated in 14-th century Europe. This method of estimation allowed them to for example, find the distance Jupiter had travelled in an certain amount of time.

2.4. Ancient of Chinese Mathematics

It was born out of many centuries of development and continued to evolve over six thousand years and still counting. An analysis of Chinese mathematics has demonstrated its unique development compared to other parts of the world, leading scholars to assume an entirely independent development. The oldest extant mathematical texts from China is Zhoubi Suanjing, variously dated to between 1200 BC and 100 BC, though a date of about 300 BC during the Warring States Period appears reasonable. However, the Tsinghua Bamboo Slips, containing the earliest known decimal multiplication table (although ancient Babylonian's had ones with a base of 60),is dated around 305 BC and or perhaps the oldest surviving mathematical texts of China.

Chinese mathematics has a strong emphasis on practical applications. A they developed important disciplines in mathematics, including Geometry, Algebra and Trigonometry. Chinese mathematicians were also the first calculate the value of the number [π] to five decimal places.

Another important invention of Chinese mathematics is the abacus, a counting frame, or a calculating tool that has been used since ancient times and which is still used in many parts of the world as we speak. Chinese mathematician also developed the I-ching, a technique for divination and astrological calculation. Divination is the practice of seeking knowledge of the future or the unknown by supernatural means.

The influence of Chinese mathematics can still be felt in modern mathematics. For instance, the concept of negative numbers was first proposed by the Chinese mathematician Yang Hui in the 13-th century. The Yin-Yang symbol, the symbol of duality is also a creation of Chinese mathematicians.

The oldest extant work on geometry in china comes from the philosophical Mohist canon c.330 BC, compile by the followers of Mozi (470-390 BC).The Mo Jing describes various aspects of many fields associated with physical science and provide a small number of geometrical theorems as well. It also defined the concepts of circumference, diamete, radius and volume.

2.5. Ancient of Indian Mathematics

The earliest civilization on the Indian subcontinent is the Indus Valley Civilization (mature second phase:2600 to 1900 BC) that flourished in the Indus river basin. Their cities were laid out with geometric regularity, but no known mathematical documents survive from this civilization.

The oldest extant mathematical records from India are the Sulba Sutras (dated variously between the 8-th century BC and the 2nd century AD), appendices to religious texts which give simple rules for constructing altars of various shapes, such as squares, rectangles, parallelograms and others. As with Egypt, the preoccupation with temple functions points to an origin of mathematics in religious ritual. The Sulba Sutras give methods for constructing a circle with approximately the same area as a given square, which imply several different approximations of the value of π. In addition, they compute the square root of 2 to several decimal places, list Pythagorean triples and give a statement of the Pythagorean theorem. All of these results are present in Babylonian mathematics, indicating Mesopotamian influence. It is not known to what extent the Sulba Sutras influenced later Indian mathematicians. As in China, there is a lack of continuity in Indian mathematics; significant advances are separated by long periods of inactivity.

Panini (c.5-th century BC) formulated the rules for Sanskrit grammar. His notation was similar to modern mathematical notation and used met rules, transformation and recursion. Pingala (roughly 3rd -1st centuries BC) in his treatise of prosody uses a device corresponding to a binary numeral system. His discussion of the combinatory of meters corresponds to an elementary version of the Binomial theorem. Pingala's work also contains the basic ideas of Fibonacci number (called matrameru).

The next significant mathematical documents from India after the Sulba Sutras are the Siddantas, astronomical treatises from the 4th and 5th centuries AD (Gupta period) Showing strong Hellenistic influence. They are significant in that they contain the first instace of trigonometric relations based on the half-chord, as is the case in modern trigonometry, rather than full chords, as was the case in Ptolemaic trigonometry. Through a series of translations errors, the word "sine" and "cosine" derive from the Sanskrit "jiya"and "kojiya".

Around 500 AD, Aryabhata wrote the Aryabhatiya, a slim volume, written in verse, intended to supplement the rules calculation used in astronomy and mathematical mensuration, though with no feeling for logic or deductive methodology. It is in the Aryabhatiya that the decimal place-value system first appears. Several centuries later the Muslim Mathematician Abu Rayhan Biruni described the Aryabhatiya as a "mix of common pe bibles and costly crystals".

In the 7th century, Brahmagupta identified the Brahmagupta theorem, Brahmagupta's identity and Brahmagupta's formula and used for the first time, in Brahma-sphuta-siddhanta, he lucidly explained the use of zero as both a placeholder and decimal digit and explained the Hindu-Arabic numeral system. It was from a translation of this Indian texts on mathematics (c.770) that Islamic mathematicians were introduced to this numeral system, which they adapted as Arabic numeral system. Islamic scholars carried knowledge of this number system to Europe by the 12th century and it has now displaced all older number system throughout the world. Various symbol sets are used to represent numbers in the Hindu-Arabic numeral system, all of which evolve from the Brahmi numerals.In the 10th century, Halayudha's commentray on Pingala's work contains a study of Fibbonacci Sequence and Pascal's triangle.

In the 12th century, Bhaskara II, who lived in southern India, wrote extensively on all then known branches of mathematics. His work contains mathematical objects equivalent or approximately equivalent to infinitesimals, the mean value theorem and the derivative of the sine function although he did not develop the notion of derivative. In the 14th century,Narayana Pandita completed his Ganita Kaumudi.

Also in the 14th century,Madhava of Sangamagrama,the founder of Kerela school of mathematics,found the Madhava-Leibniz series and obtained from it a transform series, whose first 21 terms he used to compute the value of π as 3.14159265359. Madhava also found the Madhava-Gregory series to determine the arctangent, the Madhava-Newton power series to determined sine and cosine and Taylor approximation for sine and cosine functions.In the 16^{th} century, Jyesthadeva consolidated many of the Kerela school's developments and theorems in the Yukti-bhasa. It has been argued that certain ideas of calculus like infinite series and Taylor series for some trigonometry functions, were transmitted to Europe in the 16^{th} century via Jesuit missionaries and traders who were active around the ancient port of Muziris at the time and as a result, directly influenced later European developments in analysis and calculus. However, other scholars argued that the Kerela School did not formulate a systematic theory of differentiation and integration and that there is not any diect evidence of their results being transmitted outside kerela.

2.6. Ancient of Maya Mathematics

In the pre-Columbian Americas, the Maya civilization that flourished in Mexico and central America during 1^{st} millennium AD developed a unique tradition of mathematics that ,due to its geographic isolation was entirely independent of existing European, Egyptian and Asian Mathematics. Maya numerals used a base of twenty, the vigesimal system, instead of a base of ten that forms the basis of the decimal system used by most modern cultures. The Maya used mathematics to create the Maya calendar as well as to predict astronomical phenomena in their native Maya astronomy. While the concept zero had to be inferred in the mathematics of many contemporary cultures, the Maya developed a standard symbol for it.

CHAPTER-3

GREEK MATHEMATICS AND ITS HELLENISTIC CONTRIBUTIONS

3.1. Introduction

Greek mathematics is the 5^{th} and most influential branch of mathematics in today's world. Greek mathematics was responsible for the introduction of many fundamental mathematical concepts and theorems such as the Pythagorean theorem and the Pythagorean triangle, Euclid's elements and the concept of formal proof.

A few of the most notable Greek mathematicians are:

(i) **Euclid (ca.325 BC):** Is considered to be one of most influential ancient mathematicians, Euclid is the founder of the influential "Elements of mathematics" and is of ten times referred to as the "father of geometry".

(ii) **Apollonius of Perga (ca.262 BC):** Is known for his work on conic section, ellipses and parabolas, as well as his generalization of the Pythagorean theorem.

(iii) **Archimedes of Syracuse (ca.287 BC-212 BC):** Is a mathematician of Greece, he made numerous ground breaking developments in the field of mathematics, natural sciences and Engineering. He is best known for the " Archimedes Principle" ,the law of levers and the "Archimedes theorem".

(iv) **Pythagoras (ca.570 BC-40 BC):** The founder of the "rule of Pythagoras Theorem". Pythagoras theorem states that "In a right angled triangle, square of hypotenuse side is equal to the sum of squares of the other two sides" the side of this triangle have been named perpendicular, base and hypotenuse.

3.2. Hellenistic and early Roman Period

Ancient Greek mathematics reached its acme during the Hellenistic and early roman period. Alexander the great conquest of the Eastern Mediterranean, Egypt, Mesopotamia ,the Iranian Plateau, Central Asia and parts of India led to spread the Greek culture and language across this the regions. Koine Greek became the Lingua franca of scholarship throughout the Hellenistic world and the mathematics of the classical period merged with Egyptian and Babylonian mathematics to give rise to Hellenistic mathematics.

3.2.1. Geometry

During the Hellenistic age, three construction problems in geometry became famous: doubling the cube, trisecting an angle and squaring the circle, all of which are now known to be impossible with a straight edge and compass. Many attempts were made using neusis construction including the Cissoid of Diocles, Quadratix and the Conchoid of Nicomedes. Regular polygonal had already been known before Euclid's Elements, but Archimedes extended their study to include semiregular polyhedra, also konown as Archimedean Solids.

Most of the works that became part of a standard mathematical curriculum in late antiquity were composed during the Hellenistic period: Data and Porisms by Euclid, several works by Apollonius of perga including cutting off and ratio, cutting off an area, Determinate section, Tangencies and Neusis and several works dealing with Loci, including Plane loci and conics by Apollonius, Solid loci by Aristaeus the Elder, Loci on a surface by Euclid and on means by Eratosthenes of Cyrene.

3.2.2. Applied Mathematics

Astronomy was considered one of the mathematical and accordingly many Hellenistic mathematicians devoted time to astronomy. The development of trigonometry as a synthesis of Babylonian and Greek methods is commonly attributed to Hipparchus, whose only extant work is his commentary on the phaenomena of Eudoxus and Aratus.

In the 2nd century AD, Ptolemy wrote the mathematical syntaxes, now known as the Almagest, explaining the motions of the stars and planets according to a geometric model and calculated chord tables to a higher degree of precision, along with an instruction manual, in the Handy Tables.

The ancient Greeks regarded harmonics as the science of the arrangements of pitched sounds behind musical melody, including the principles which govern them. It was the most important branch of ancient Greek musical theory, studied by Philosophers, mathematicians and astronomers as well as by musical specialists. Works in mathematical harmonic in the Hellenistic period include the Sectio Canonis, attributed to Euclid and Ptolemy's Harmonics.

CHAPTER-4

MATHEMATICS IN THE ISLAMIC GOLDEN AGE

4.1. Introduction

The Islamic empire established across the Middle East, Central Asia, North Africa, Iberia, and in parts of India in the 8^{th} century made significant contributions towards mathematics. Although most Islamic texts on mathematics were written in Arabic, they were not all written by Arabs, since much like the status of Greek in the Hellenistic world, Arabic was used as the written language of non-Arab scholars throughout the Islamic world at the time.

4.2. Islamic Mathematics

In the 9^{th} century, the Persian mathematician Muhammad Lbn Musa al-Khwarizmi wrote an important book on the Hindu-Arabic Numerals and one on the methods for solving equations. His book on the calculation with Hindu numerals, Written about 825 along with the work of AL-Kindi, were instrumental in spreading Indian mathematics and Indian numerals to the west. The word algorithm is derived from the Latinization of his name, Algorithmi and the word algebra from the title of one of his works, AI-Kitab al-mukhtasar fI hisab al-gabr wa'l-muqabala. He gave an exhaustive explanation for the algebraic solution of quadratic equations with positive roots and he was the first to teach algebra in an elementary form and for its own sake. He also discussed the fundamental method of “reduction”and “balancing”,reffering to the transposition of subtracted terms to the other side of an equation, that is, the cancellation of like terms on opposite sides of the equation. This is the operation which al-khwarizmi originally described as aI-jabr. His algebra was no longer concerned “with a series of problem to be resolved, but an exposition which starts with primitive terms in which the combinations must give all possible prototypes for equations, which henceforward explicitly constitute the true object of study”.

In Egypt,Abu Kamil extended algebra to the set of irrational number accepting square roots and fourth roots as solutions and coefficients to quadratic equations. He also developed techniques used to solve three nonlinear simultaneous equations with three unknown variables. One unique feature of his work was trying to find all the possible solutions to some of his problems, including one where he found 2676 solutions.His works formed an important foundation for the development of algebra and influenced later mathematicians, such as al-Karaji and Fibonacci.

Further developments in algebra were made by AI-Karaji in his treatise al-Fakhri, Where he extends the methodology to incorporate integer powers and integer roots of unknown quantities. Something close to a proof by mathematical induction appears in a book written by AI-Karaji around 1000 AD, who used it to prove the binomial theorem, Pascal’s triangle and the sum of integral cubes. The historian mathematics, F.Woepcke, praised AI-Karaji for being “the first who introduced the theory of algebraic calculus”. Also in 10^{th} century, Abul Wafa translated the works of Diophantus into Arabic.lbn al-Haytham was the first mathematician to derive the formula for sum of the fourth powers, using a method that is readily generalizable for determining the general formula for the sum of any integral powers. He performed an integration in order to find the volume of a Paraboloid and was able to generalize his result for the integrals of Polynomials up to the fourth degree. He thus came close to finding a general formula for the integrals of polynomials, but he was not concerned with any polynomials higher than the fourth degree.

In the late 11^{th} century, Omar Khayyam wrote Discussions of the Difficulties in Euclid, a book about what he perceived as flaws in Euclid's Elements, especially the parallel postulate. He was also the first to find the general geometric solution to cubic equations.

In the 13^{th} century, Nasir al-Din Tusi (Nasireddin) made advances in spherical trigonometry. He also wrote influential work on Euclid's parallel postulates. In the 15^{th} century, Ghiyath al-Kashi computed the value of π to the 16^{th} decimal places. Kashi also had an algorithm for calculating n-th roots, which was special case of the methods given many centuries later by Ruffini and Horner.

Other achievements of Muslim mathematicians during this period include the addition of the decimal point notation to the Arabic numerals, the discovery of all the modern trigonometric functions besides the sine, al-Kindi's introduction of cryptanalysis and frequency analysis, the development of analytic geometry by lbn al-Haytham, the begininig algebraic geometry by Omar Khayyam and the development of an algebraic notation by al-Qalasadi.

During the time of the Ottoman Empire and Safavid Empire from the 15^{th} century, the development of Islamic mathematics became stagnant.

CHAPTER-5

EUROPEAN RENAISSANCE AND THE BIRTH OF MODERN MATHEMATICS.

5.1. Introduction

The European Renaissance (14^{th} -17^{th} centuries) played a crucial role in the developments of modern mathematics, with a renewed interest in classical Greek and Roman texts and the invention of the printing press facilitating the spread of knowledge. while the formal theory of matrices emerged in the 19^{th} century, the renaissance laid the groundwork with advancements in algebra, geometry and the application of mathematics to art and science.

The renaissance, meaning “rebirth”, began in Italy and spread across Europe. It was inspired by a renewed interest in the classical knowledge of Ancient Greece and Rome and focused on:

. Humanism (imoortace of human reasoning and experience).

. Scientific inquiry and empirical observation.

. Artistics and architectural advancements.

. Rediscovery of ancient texts via Arabic translations.

5.2. Mathematics Before the Renaissance.

Before the Renaissance,much of Europe's mathematical knowledge came from:

.Greek sources like Euclid,Pythagoras and Archimedes.

.Arabic scholars such as AI-Khwarizmi (algebra), Omar Khayyam and AI-Tusi.

. The Hindu-Arabic nemeral system,Which was not yet widely accepted in Europe.

Mathematics in medieval Europe was mostly theoritical, tied to philosophy or theology with little focus on practical applications.

5.3. Renaissance Contribution's to Modern Mathematics.

(i) Revival of Classical Knowledge.

. Ancient Greek and Arabic mathematical texts were rediscovered, translated and studied.

. The works of Euclid, Archimedes and Ptolemy were reintroduced to scholars.

(ii) Adoption of Hindu-Arabic Numerals.

. Replacing Roman numerals, theMHindu-Arabic system (including zero) made calculations easier.

. Promoted by figures like Leonardo of Pisa (Fibonacci) through words like Liber Abaci (1202).

(iii) Algebra Development.

. Algebra evolved from rhetorical (word-based) to symbolic form.

. Francois Viete introduced the first systemetic use of symbols for known and unknown quantites.

. This was a major shift towards abstract thinking in mathematics.

(iv) Analytic Geometry..

. Rene Descartes (1596-1650) merged algebra with geometry in his La Geometric,leadind to coordinate geometry.

. This laid the foundation for calculus and modern physics.

(v) Perspective in Art.

. Artists like Leonerdo da Vinci and Albrecht Durer used mathematical principles to create realistic art.

. This encouraged study in geometry and proportions.

(vi) Invention of the printing Press.

. Allowed mathematical books and ideas to spread widely and rapidly.

. Helped standardize mathematical notation and terminology.

(vii) Rise of Scientific Inquiry.

. Mathematics became central to the Scientific Revolution, which followed the Renaissance.

. Scientists like Galileo Galilei and Johannes Kepler used mathematics to describe natural laws.

CHAPTER-6

MATHEMATICS DURING THE SCIENTIFIC REVOLUTION

6.1. Introduction

The scientific revolution (roughly 1543-1700) was a pivotal period in the development of modern science and mathematics. It marked a dramatic shift from medieval views based on authority and tradition to a new emphasis on observation, experimentation and rational analysis. Mathematics played a central role in this transformation.

6.2. Mathematics in the 17th century

The 17thcentury saw an unprecedented increase of mathematical and scientific ideas across Europe. Tycho Brahe had gathered a large quantity of mathematical data describing the positions of the planets in the sky. By his position as Brahe's assistant, Johannes Kepler was first exposed to an seriously interacted with the topic of planetary motion. Kepler's calculations were made similar by the contemporaneous invention of Logarithms by John Napier and Jost Burigi. Kepler succeeded in formulating mathematical laws of planetary motion. The analytic geometry developed by Rene Descartes (1596-1650) allowed those orbits to be plotted on a graph, in Cartesian coordinates.

Building on earlier work by many predecessors, Issac Newton discovered the laws of physics that explain Kepler's Lawa and brought together concepts now known as Calculus. Independently, Gottrified Wilhelm Leibniz, developed calculus and much of the calculus notation still in use today. He also refined the binary number system, which is the foundation of nearly all digital (electronic,solid-state,discrete logic) computers.

In addition to the application of mathematics to the studies of the heavens applied mathematics began to expand into new areas, with the correspondence of Pierre de Fermat and Blaise Pascal. Pascal and Fermat set the groundwork of the investigations of Probability theory and the corresponding rules of combinatory in their discussions over a game of gambling. Pascal, with his Wager, attempted to use the newly developing probability theory to argue for a life devoted to religion, on the grounds that even if the probability of success was small the rewards were infinite.

6.3. Mathematics in the 18th century

The most influential mathematician of the 18th century was arguably Leonhard Euler (1707-83).His contributions range from founding the study of graph theory with the Seven Bridges of Konigsberg problem to standardizing many modern mathematical terms and notations. For example, he named the square root of minus 1 with the symbol I, and he popularized the use of the Greek letter π to stand for the ratio of the circle's circumference to it's diameter. He made numerous contributions to study of topology, graph theory, calculus, combinatory and complex analysis as evidenced by the multitude of theorems and notatios named from him. Other important European mathematician of the 18th century included Joseph Louis Lagrange, who did pioneering work in number theory, algebra differential calculus and the calculus of variations and Pierre-Simon Laplace.

CHAPTER-7

MODERN 19^{TH} CENTURY MATHEMAICS: A PERIOD OF FORMALIZATION AND EXPANSION

7.1. Introduction

The 19^{th} century marks a transformative era in the history of mathematics, often referred to as the beginning of modern mathematics. This period was characterized by rigorous formalization, the birth of entirely new fields and the global expansion of mathematical thought beyond its classical roots in geometry and calculus.

1. **Rise of Rigor and Foundations.**

Prior to the 19^{th} century, many mathematical results were intuitive or based on informal reasoning. However, mathematicians like Augustin-Louis Cauchy initiated a rigorous foundation for calculus, introducing formal definitions of limits, continuity and convergence. This laid to groundwork for real analysis, which was later advanced by Karl weiestrass, who eliminated reliance on geometric intuition.

The development of rigorous logic and set theory, especially through the work of Georg Cantor, revolutionized the foundations of mathematics. Cantor's theory of infinite sets introduced the concepts of cardinality and distinguished between different sizes of infinity, challenging and expanding traditional mathematical view.

2. **Expansion into Abstract Algebra and Number theory.**

The 19th century saw a shift toward more abstract and general framework. Evariste Galois developed group theory, which became fundamental to the study of algebraic structures. His insight into the solvability of polynomial equations laid the foundations of abstract algebra.

Meanwhile, Carl Friedrich Gauss significantly advanced number theory through his work Disquisition's Arithmetician. His contributions, along with those of Dirichlet, Legendre and Riemann, led to deeper understanding of prime numbers,quadratic reciprcity and the distribution of primes.

3. Geometry Beyond Euclid.

The long-standing dominance of Euclidean geometry was challenged in the 19th century with the emergence of non-Euclidean geometries. Mathematicians like Nikolai Lobachevsky, Janos Bolyai and later Bernhard Riemann explored geometries in which Euclid's fifth postulates (the parallel postulates) did not hold. This opened the door to hyperbolic, elliptic and Riemannian geometry, which would later become crucial to Einstein's theory of relativuty.

4. Development of Mathematical Logic and Set Theory.

The late 19th century also witnessed the birth of mathematical logic. Thinkers such as George Boole(Boolean algebra) and Gottolob Frege made early advances that would influence the formalization of logic and the eventual development of computability theory and mathematical foundations in 20th century.

Cantor's set theory, while revolutionary, also sparked philosophical debates that would eventually lead to foundational crises and the formalist, logistic and intuitionist schools of thought.

5. Mathematical Analysis and the infinite.

Mathematicians like Riemann, Weirstrass and Debekind refined the concepts of functions and the structure of the real number line. Riemann's ideas about integrals and manifolds broadened the scope of analysis and laid foundations for modern differential geometry.

6. Mathematics and physics interconnection.

The 19th century also strengthened the ties between mathematics and physics. The development of Partial differential equations, Fourier analysis and Vector calculus was deeply tied to problems in heat, electromagnetism and fluid dynamics. James Clerk Maxwell's equations describing electromagnetism are a striking example of how mathematical structures describe physical reality. Computers science, artificial intelligence and data analytics are heavily impact on our

CHAPTER-8

MATHEMATICS IN THE 20TH AND 21ST CENTURIES

8.1. Introduction

The 20th and 21st centuries represents the period of growth and transformation in mathematics. In 20th century, Mathematics Undergo a foundational shift. The original work in logic and set theory by David Hilbert, Bertrand Russell and Kurt Godel are reshaped the philosophical basis of mathematics. The new fields are in the same time such as topology, functional analysis, abstract algebra and category theory appeared and matured. In this century the application of mathematics in physics, engineering and economics, particularly after the world wars, leading the growth of applied mathematics, operations research and numerical analysis.

In 21st century, mathematics is very advance in mathematical theories and methods.

8.2. The Age of Abstraction and Computation

(i) Rise of abstract mathematics: In 20th century the new fields are abstract algebra, category theory, topology and functional analysis became central. Set theory, formalized by Georg Cantor in 19th century became foundational for all mathematics. David Hilbert emphasized the correctness of the idea of that mathematics should be built upon a complete and consistent set of axioms.

(ii) Computational revolution: Development of the computers in the middle of 20th century conducted to a second pillar in this period: computation.

- Alan Turing is situated the theoretical base with the concept of Turing machine (1936).

- In 1940's invention of the the digital computers are revolutionized the applied mathematics and also number theory, combinatory and algorithmic analysis.

- Computational complexity theory was developed to classify the problems based on the resources like time or memory are required to solve them.

(iii) Interdisciplinary Growth:

- In physics**,** abstract mathematical structure group theory become the central in quantum mechanics and particle physics.

- Biology, computer science and economics are integrated game theory, algorithm design and mathematical modelling.

- Cryptography was developed from the classical ciphers to complex algorithms rooted in number theory and elliptic curve are determined by the need of digital security.

(iv) Formal Proof and Automation:

The first theorem, four colour theorem (1976) was proved by using computer. Recently, the automated theorem was proved by Coq, Lean and Isabelle are being used to verify the complex theorems with machine precision.

(v) Big Data and Mathematical Modelling:

In 21st century has more featured the computational nature of modern mathematics. Data science, machine learning and AI are dependent on linear algebra, statistic and optimization. Mathematical biology, epidemiology and climate science used in differential equation to model complex system **.**

CONCLUSION

The History of Mathematics is the development of that the progress of humanity across different civilizations and periods. From ancient counting system and geometric perceptions in Mesopotamia, Egypt and India to the severe logic of Greek mathematics, the discipline of growth through contributions from every part of the world.

It developed the mathematical thought through the historical periods and including the Islamic golden age, the European renaissance, scientific revolution and the 17^{th} to 21^{st} centuries includes the development of calculus, geometry, logic, abstraction, computation and interdisciplinary applications.

Analyzing the cultural, philosophical and scientific contexts in which the mathematics were developed. This historical journey developed the human understanding the how cultural, scientific and technological growth in mathematics in modern life.

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